The Gravitational Coherence Surface

Signal-to-Noise Limits on Gravitational Identity

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Abstract

Every gravitating system generates an exterior field whose multipole structure carries information about the source's internal mass distribution. At sufficient distance, each mode of that field becomes difficult to distinguish from the gravitational environment in which the source is embedded. We define the *gravitational coherence surface* as the locus where a chosen source mode falls to parity with a calibrated background noise floor. The resulting object is an operational boundary of gravitational distinguishability—not a dynamical stability boundary—and is defined entirely within standard general relativity and its weak-field Newtonian limit.

We derive single-crossing conditions guaranteeing that each mode-defined shell is well-posed, give conditional nesting criteria for ordinary exterior sources, and present an inversion formula recovering the observable exterior multipole spectrum from the coherence geometry. We compute illustrative coherence structures for the Sun, Earth, Jupiter, and the Milky Way, and show that the formalism reproduces the anomalous gravitational status of the Moon as a consistency check: the Moon is dynamically Earth-bound while lying outside Earth's monopole coherence shell. To our knowledge, the coherence surface is a gravitational distinguishability boundary that is simultaneously multipole-resolved, defined by signal-to-noise rather than force balance or orbital stability, and explicitly sensitive to the surrounding gravitational background. In the dominant-neighbor limit, its monopole shell reproduces an acceleration-parity-style sphere-of-influence criterion; the higher-l shells appear to be new objects within the usual influence-boundary literature. We outline an observational program centered on environmental dependence of recoverable gravitational extent at fixed internal mass model and provide a compact reproducibility appendix for the worked examples.

1 Motivation

The Moon orbits Earth, yet the Sun's gravitational acceleration at the Moon is larger than Earth's by a factor of approximately two. The standard resolution is dynamical: bound motion in the three-body problem depends on differential forcing and orbital stability, not on whichever body provides the larger absolute acceleration. The Hill sphere and related spheres of influence capture that fact well.

But there is a different question hiding in the machinery. At the Moon's location—or more generally at some distance from a source—can an observer still *read* that source's gravitational identity against the surrounding gravitational clutter? Not merely whether the source still exerts force, but whether a particular part of its field remains cleanly attributable to that source rather than to the background. That is the question addressed here.

The gravitational coherence surface is the operational boundary at which a chosen source signal falls to parity with a calibrated noise floor. It is closer in spirit to an observability threshold than to a stability boundary. Matter crosses it freely. Orbits need not notice it sharply. What changes is the external readability of source-specific structure.

In the simple single-neighbor estimate developed below, Earth's monopole coherence radius lies at approximately 261,000 km, inside the Moon's orbit at 384,000 km. The Moon is dynamically part of the Earth system while lying outside Earth's identity-coherence shell. That is not a contradiction. It is precisely the point: gravitational distinguishability and dynamical binding are related, but they are not the same creature.

2 Setup and Notation

2.1 The Source

Let S be a finite, isolated, quasi-stationary gravitating source of total mass M with center of mass at the origin. In the weak-field regime, the exterior Newtonian potential admits the multipole expansion

$$\Phi(r,\theta,\phi) = -\frac{GM}{r} - G\sum_{l\geq 2}\sum_{m}\frac{Q_{lm}Y_{lm}(\theta,\phi)}{r^{l+1}},$$

where Q_{lm} are the mass multipole moments,

$$Q_{lm} = \int_S \rho(\mathbf{r}')\, r'^l Y^*_{lm}(\theta',\phi')\, d^3r',$$

and the dipole term ($l=1$) vanishes in the center-of-mass frame.

2.2 The Tidal Field

For $l \geq 2$, the physically relevant local observable is the tidal field, i.e. the electric part of the Weyl tensor, which in the Newtonian limit is

$$E_{ij}(\mathbf{r}) = -\partial_i\partial_j\Phi.$$

We package the lth tidal contribution into a scalar amplitude

$$T_l(r,\hat{n}) = \alpha_l\,\frac{GQ_l}{r^{l+3}},$$

where $Q_l = \sqrt{\sum_m |Q_{lm}|^2}$ is the total multipole amplitude at order l and α_l is an order-unity geometric coefficient encoding angular structure.

2.3 The Monopole Channel

The monopole requires separate treatment. A freely falling observer can transform away a uniform local acceleration but not tidal curvature. Accordingly, the monopole is not an invariant local scalar in the same sense as the higher tidal channels.

We therefore define the $l = 0$ channel operationally: one specifies either

1. an asymptotically inertial frame together with an ephemeris model, or
2. a finite-baseline tracking protocol that turns the source's monopole field into a measurable residual acceleration relative to background expectations.

This paper keeps that distinction explicit rather than treating the monopole and higher multipoles as identical objects.

2.4 The Background and Noise Hierarchy

The source is embedded in an environment containing other gravitating systems. We distinguish two noise notions from the outset.

Environmental floor. For $l \geq 2$, a stochastic tidal background $N_{\text{env}}(\hat{n})$. For $l = 0$, a background acceleration amplitude $a_{\text{bg}}(\hat{n})$. For a single dominant neighbor of mass M' at distance D,

$$N_{\text{env}} \approx \frac{GM'}{D^3}, \qquad a_{\text{bg}} \approx \frac{GM'}{D^2}.$$

Measurement-aware effective floor. Including instrumental and modeling uncertainties,

$$N_{\text{eff}}^2 = N_{\text{env}}^2 + N_{\text{inst}}^2 + N_{\text{model}}^2,$$

$$a_{\text{eff}}^2 = a_{\text{bg}}^2 + a_{\text{inst}}^2 + a_{\text{model}}^2.$$

The environmental quantities define the fundamental coherence geometry; the effective quantities define the observable coherence geometry relevant to data analysis. When instrumental and modeling terms are neglected, $N_{\text{eff}} \to N_{\text{env}}$, recovering the fundamental surfaces.

3 Definition of the Coherence Surface

3.1 The Coherence Radius

Definition 1 (Visibility ratio). For $l \geq 2$, define

$$\rho_l(r, \hat{n}) = \frac{T_l(r, \hat{n})}{N_{\text{eff}}(r, \hat{n})}.$$

For the monopole, with a specified frame or tracking protocol,

$$\rho_0(r, \hat{n}) = \frac{a_0(r)}{a_{\text{eff}}(r, \hat{n})}, \qquad a_0(r) = \frac{GM}{r^2}.$$

Definition 2 (Coherence radius). The coherence radius for mode l in direction $\hat{n}$ is the solution of $\rho_l = 1$:

$$r_l(\hat{n}) = \left(\frac{\alpha_l G Q_l}{N_{\text{eff}}(\hat{n})} \right)^{1/(l+3)} \qquad (l \geq 2),$$

$$r_0(\hat{n}) = \left(\frac{GM}{a_{\text{eff}}(\hat{n})} \right)^{1/2} \qquad (l = 0).$$

3.2 The Coherence Surface and Full Structure

Definition 3 (Coherence surface). The gravitational coherence surface at order l is

$$\Sigma_l = \{ r_l(\hat{n})\, \hat{n} \mid \hat{n} \in S^2 \}.$$

Definition 4 (Complete coherence structure). The full coherence structure of S is the family

$$\mathcal{C}(S) = \{\Sigma_l\}_{l=0}^{\infty},$$

subject to the nesting conditions established below.

4 Properties

4.1 Single-Crossing Condition

Definition 5 (Visibility flow). The logarithmic radial derivative of the visibility ratio is

$$\beta_l(r, \hat{n}) = \frac{d \ln \rho_l}{d \ln r}.$$

For $l \geq 2$,

$$\beta_l = -(l+3) - \frac{d \ln N_{\text{eff}}}{d \ln r},$$

and for $l = 0$,

$$\beta_0 = -2 - \frac{d \ln a_{\text{eff}}}{d \ln r}.$$

Proposition 1 (Single crossing). If $d \ln N_{\text{eff}} / d \ln r > -(l+3)$ along a radial ray for $l \geq 2$, then $\beta_l < 0$, the visibility ratio is strictly decreasing, and $\rho_l = 1$ has at most one solution on that ray. Likewise, if $d \ln a_{\text{eff}} / d \ln r > -2$, the monopole shell is single-valued.

Proof. Under the stated conditions, $\beta_l < 0$ (respectively $\beta_0 < 0$), so the corresponding visibility ratio is monotonically decreasing. A monotone function crosses a fixed threshold at most once. □

The condition requires only that the background not fade outward faster than the source signal itself. For any approximately homogeneous or slowly varying background field, this is easily satisfied.

4.2 Conditional Nesting

The statement that higher multipoles decohere closer to the source is physically sensible but not logically free.

Proposition 2 (Nesting condition). Let $A_l = \alpha_l G|Q_l|$. For fixed local background, the coherence radii satisfy $r_{l'} < r_l$ for $l' > l$ if and only if

$$A_{l'}^{1/(l'+3)} < A_l^{1/(l+3)}.$$

For ordinary sources with $Q_l \sim Md^l$ (where d is a characteristic source size), this reduces to the exteriority condition $d < r_l$, requiring the coherence shell to lie outside the source. That condition is trivially satisfied for all physical systems in the weak-field regime. In every worked example here (Sun, Earth, Jupiter, Milky Way), nesting holds comfortably. Pathological moment hierarchies could violate nesting, but they are not expected for ordinary finite sources with bounded extent and nonpathological moment scaling.

4.3 Relation to the Hill Sphere

In a single-neighbor monopole estimate,

$$r_0 = D\left(\frac{m}{M}\right)^{1/2}, \qquad r_H = D\left(\frac{m}{3M}\right)^{1/3}.$$

Therefore,

$$\frac{r_0}{r_H} = 3^{1/3}\left(\frac{m}{M}\right)^{1/6}.$$

This is exactly what one should expect if coherence loss is an observability threshold and the Hill sphere is a dynamical stability threshold.

Table 1 summarizes the ratio for representative systems.

Table 1: Ratio of monopole coherence radius to Hill sphere radius. For small mass ratios the coherence shell lies well inside the Hill sphere. For comparable masses the two become similar.

System	m/M	r_0/r_H
Earth / Sun	3.0×10^{-6}	0.17
Jupiter / Sun	9.5×10^{-4}	0.45
Moon / Earth	1.2×10^{-2}	0.69
Sun / Milky Way	2.0×10^{-11}	0.024
Milky Way / Andromeda	6.7×10^{-1}	1.35
Globular cluster / Milky Way	1.0×10^{-6}	0.14

4.4 Environmental Dependence and Anisotropy

Because N_{env} and a_{bg} depend on the gravitational neighborhood, the coherence surface is not an intrinsic property of the source. A $10^6 M_\odot$ globular cluster has monopole coherence radius

- in the Milky Way disk (nearest massive neighbor at approximately 8 kpc): $r_0 \approx 36$ pc,
- in a void (nearest neighbor at approximately 5 Mpc): $r_0 \approx 16$ kpc,
- giving an expansion factor of approximately $442\times$.

Same object, same mass—but its gravitational identity extends hundreds of times farther in empty space. Directional anisotropy in the background compresses the shell toward strong perturbers and elongates it away from them. The resulting shape carries information about the environment as much as about the source.

4.5 Conditional Inversion

If the background field is independently calibrated and the angular coefficients are known, one may recover the observable exterior multipole amplitudes:

$$|Q_l| = \frac{N_{\text{eff}}(\hat{n})\, r_l(\hat{n})^{l+3}}{\alpha_l G}.$$

The leading logarithmic uncertainty is

$$\delta \ln |Q_l| = (l+3)\, \delta \ln r_l + \delta \ln N_{\text{eff}} + \delta \ln \alpha_l,$$

showing that higher-l moments are more sensitive to radius errors.

This statement must be read honestly. The inversion recovers the observable exterior multipole spectrum up to resolved order under calibrated assumptions. It does **not** uniquely reconstruct arbitrary interior microstructure, because distinct interiors can share the same exterior low-order moments. The coherence geometry encodes the recoverable exterior spectrum, not a one-to-one reconstruction of all interior detail.

5 Static Distinguishability

5.1 Why Not Shannon Capacity

Shannon channel capacity is a rate statement tied to bandwidth, coding, and integration time. A static or quasi-static field comparison at one scale does not yet provide that structure. Rather than claim a theorem not earned, we define a static distinguishability functional appropriate to the quasi-static gravitational-field problem.

5.2 The Distinguishability Functional

For each mode, let $g_0 = 1$ and $g_l = 2l + 1$ for $l \geq 2$. Define

$$D_l(r) = g_l \max\left[\log_2 \rho_l^{\text{obs}}(r),\, 0\right],$$

$$D(r) = \sum_l D_l(r).$$

$D(r)$ is dimensionless. It is not a communication rate and should not be read as bits per second. It is a logarithmic visibility budget: how far above threshold each recoverable mode sits, weighted by angular degeneracy. Larger D means a richer menu of source-specific structure remains distinguishable at radius r. When all channels fall below threshold, $D = 0$.

One may also define an effective class count

$$\Omega_{\text{eff}}(r) = 2^{D(r)}.$$

This is not a fundamental state count and not a quantization claim. It summarizes how many source configurations are externally distinguishable under the adopted thresholding, and is operationally meaningful for model selection: at distance r, Ω_{eff} estimates how many source models can be discriminated from their gravitational signatures against the noise.

5.3 Toward a True Information Rate

A genuine information-rate theory becomes possible for time-dependent sources. If a mode varies at frequency f and an experiment has bandwidth Δf and integration time τ, one can build a real communication problem with a detector noise spectral density and derive an information rate. That extension is deferred rather than implied. The present paper remains on the safer ground of static distinguishability.

5.4 The Sun: Illustrative Distinguishability Profile

Table 2: Distinguishability profile for the Sun in the local stellar neighborhood, using an illustrative dominant-neighbor background with $N_{\text{env}} \approx 4.5 \times 10^{-31}\,\text{s}^{-2}$ and the channel set described in Appendix A. At 1 AU, multipole structure through $l = 2$ is resolvable; beyond 100 AU, only the monopole survives.

Distance	$l_{\max}$	Modes	$D(r)$
0.1 AU	6	28	490
1 AU	2	6	130
10 AU	2	6	41
100 AU	0	1	24
1,000 AU	0	1	18
0.1 pc	0	1	9
1 pc	0	1	2
2 pc	0	1	0.2

6 Computed Examples

The numbers in this section use the simplified single-dominant-neighbor picture. They are useful for scale, not precision inference. Appendix A makes the adopted conventions explicit.

6.1 The Sun in the Stellar Neighborhood

The monopole radius (2.15 pc) is approximately half the distance to Proxima Centauri. Detailed multipolar information dies close in; the monopole survives farthest.

Mode	Encodes	Coherence radius
$l = 0$	Total mass (identity)	2.15 pc (444,000 AU)
$l = 2$	Rotational oblateness	13.1 AU
$l = 4$	Internal density layering	0.77 AU
$l = 6$	Fine radial structure	0.16 AU

6.2 Earth in the Solar System

For the monopole channel, the dominant background is the Sun's acceleration at 1 AU. For tidal channels, the dominant background is the Moon, whose tidal field at Earth exceeds the Sun's by a factor of approximately 2.2.

Mode	Encodes	Coherence radius
$l = 0$	Mass identity	41 $R_\oplus$ (261,000 km)
$l = 2$	Oblateness	7.1 $R_\oplus$ (45,000 km)
$l = 4$	Density structure	1.6 $R_\oplus$ (10,000 km)

The Moon orbits at 384,000 km, outside the monopole coherence radius. This does **not** say Earth's gravity is absent there. It says that the Earth-specific monopole signal is no longer cleanly isolated from the background under the adopted criterion. The Moon is therefore an example of dynamical membership without monopole coherence.

6.3 Jupiter in the Solar System

Mode	Encodes	Coherence radius
$l = 0$	Mass identity	0.16 AU
$l = 2$	Oblateness	0.014 AU (28 R_J)
$l = 4$	Shell structure	0.003 AU (7 R_J)

6.4 The Milky Way in the Local Group

Taking Andromeda at approximately 770 kpc as the dominant background source gives the following order-of-magnitude scales:

Mode	Encodes	Coherence radius
$l = 0$	Total mass	629 kpc
$l = 2$	Disk shape	81 kpc

The quadrupole value is best interpreted as an order-of-magnitude recoverability scale for the disk's non-spherical signature in the Local Group background. Whether a corresponding dynamical transition exists for satellite orbits is a separate question, not an automatic corollary of the shell itself.

7 Physical Interpretation and Relation to Prior Boundaries

7.1 What the Coherence Surface Is and Is Not

The coherence surface is not a horizon, not a physical barrier, and not by itself a force cutoff. It marks the distance at which a chosen source channel reaches parity with a specified noise floor. Beyond that point the source's mode becomes hard to attribute cleanly against the environment, but the field still exists and can still matter dynamically.

This distinction matters especially for torque. A quadrupole mode that lies below a background distinguishability threshold need not have literally zero dynamical influence on a satellite. It means that cleanly isolating that source-specific quadrupole against the environment is no longer feasible. Any claim that a coherence radius sets a torque cutoff requires a separate perturbative bridge theorem or numerical experiment. No such theorem is claimed here.

7.2 Survey of Prior Gravitational Influence Boundaries

The question "where does a body's gravitational influence end?" has a long history in celestial mechanics. The principal prior definitions differ in what they measure:

- **The Laplace sphere of influence (1799)** defines a patched-conic frame-switching scale, $r_{\mathrm{SOI}} = a(m/M)^{2/5}$. It is monopole-only and contains no explicit noise concept.
- **The Hill sphere (1878)** defines a dynamical stability boundary, $r_H = D(m/3M)^{1/3}$. It is likewise monopole-only.
- **The Chebotarev sphere (1964)** and its modern reinterpretations define force-balance or perturbation-balance scales.
- **The black-hole influence radius** $r_{\mathrm{inf}} = GM_{\mathrm{BH}}/\sigma^2$ is the closest classical analog to the monopole coherence shell, but it uses a velocity-dispersion proxy rather than an explicit environmental noise floor.
- **Velocity-dependent influence scales** generalize the boundary to encounter context.

7.3 What the Coherence Surface Adds

The new ingredient is not merely another radius. It is a shift in what is being bounded.

In the dominant-neighbor limit, the monopole coherence shell reproduces an acceleration-parity-style influence scale. It does **not** collapse every historical sphere-of-influence definition into one object; the Hill sphere remains the relevant dynamical stability boundary. What the coherence formalism contributes is a signal-to-noise criterion, an explicit environmental calibration, and a natural extension to higher multipoles.

Table 3: Comparison of influence-boundary definitions. The coherence surface appears to be unusual in combining mode resolution, a signal-to-noise criterion, and explicit environmental calibration within a single boundary concept.

Boundary	Criterion	Multipole-resolved?	How environment enters
Laplace SOI (1799)	Frame-switch scale	No	Through orbital geometry only
Hill sphere (1878)	Orbital stability	No	Through three-body geometry
Chebotarev-type sphere	Force or perturbation balance	No	Weakly
BH influence radius	Potential dominance	No	Partly (σ proxy)
Velocity-dependent SOI	Encounter dynamics	No	Partly (v_{enc} proxy)
Coherence surface	Signal-to-noise distinguishability	Yes (all l)	Explicitly through $N(\hat{n})$ or $a_{\mathrm{eff}}(\hat{n})$

We are not aware of direct higher-l analogs within the usual influence-boundary literature.

8 Observational Program

8.1 Environmental Dependence of Recoverable Extent

The cleanest empirical target is environmental dependence of recoverable gravitational extent at fixed source mass and fixed internal mass model. In the monopole estimate,

$$r_0 \propto \left(\frac{GM}{a_{\mathrm{eff}}}\right)^{1/2},$$

so a quieter external field should enlarge the observable identity shell.

Promising probes include satellite-galaxy distributions, weak-lensing shear profiles, stellar-stream legibility, and the radial reach over which non-axisymmetric structure can be recovered from the field model. None of these tests is uniquely diagnostic by itself. Each must control for halo mass, concentration, assembly history, stripping, and ordinary environmental processing. The distinctive claim is narrower: after such controls, the calibrated external background should still matter.

8.2 The Milky Way Quadrupole Scale

The 81 kpc quadrupole coherence radius should be read as an inference transition, not a torque cliff. Inside that scale the disk quadrupole may be recoverable more cleanly against the Local Group background; outside it, any external reconstruction of the disk-specific quadrupole should become harder. A dynamical consequence for satellite precession is plausible but remains a conjecture pending dedicated perturbative analysis or simulation.

8.3 Diffuse and Isolated Systems

Low-surface-brightness galaxies are appealing targets: isolation lowers the external floor, and extended baryonic structure can enhance low-order non-spherical moments relative to compact systems of similar mass. The prediction should be stated modestly: at fixed internal model and survey sensitivity, such systems are good candidates for enlarged recoverability windows. This is a target-selection statement first, and only secondarily a competition with conventional halo explanations.

8.4 Minimal Analysis Pipeline

A practical analysis pipeline can be stated compactly.

1. Specify an internal source model and the channel set to be tested.
2. Estimate $N_{\rm env}(\hat{n})$ or $a_{\rm bg}(\hat{n})$ from catalogs, simulations, or local perturber reconstructions.
3. Fold in instrumental and modeling uncertainties to obtain $N_{\rm eff}$ and $a_{\rm eff}$.
4. Compute $r_l(\hat{n})$, compare the inferred shells with recoverable field structure, and test whether residual recoverability trends at fixed internal model correlate with environment after standard covariates are controlled.

That is the operational bridge from definition to data. It does not prove the effect in advance, but it makes the hypothesis testable.

9 Extensions

9.1 Time-Dependent Sources

For binaries, pulsators, or driven systems, each pair (l, f) defines its own coherence condition against a frequency-dependent noise floor. That setting is where the formalism can legitimately grow into a true information-rate theory with genuine Shannon capacity.

9.2 Relativistic Generalization

In fully relativistic language one may replace

$$E_{ij} \to E_{ab} = C_{acbd} u^c u^d,$$

$$Q_{lm} \to \text{Geroch–Hansen relativistic multipole moments},$$

and

$$N \to \text{a stochastic Weyl-curvature correlator}.$$

In the weak-field limit the present definitions are recovered. A full curved-background treatment remains future work.

9.3 Hierarchical Coherence

Complex systems possess nested coherence structures: stars inside clusters, clusters inside galaxies, galaxies inside groups. The framework naturally supports this hierarchy because it is defined mode by mode and environment by environment. In plain terms, a cluster-level coherence structure can enclose nested stellar coherence structures without collapsing them into a single radius.

9.4 Deformation Budget

To track how much readable structure survives across scale, define

$$B_l(R) = g_l \int_{r_{\rm in}}^{R} \frac{dr}{r}\, \max[\log_2 \rho_l(r), 0].$$

The total

$$B(R) = \sum_l B_l(R)$$

is not a conserved quantity and not an action. It is a disciplined accounting tool: it compares a high-order channel that burns brightly but dies quickly with a lower-order channel that survives weakly across many decades in radius.

9.5 Further Literature Connections

The galactic tidal-tensor formalism provides the natural language for computing environmental tidal floors on galactic scales. The tidal Love-number literature connects the response of compact bodies to external tidal fields; the coherence surface addresses the complementary question of where those external fields themselves cease to be cleanly attributable. Gravitational-wave detection horizons are an obvious frequency-domain analog: a mode-by-mode signal is compared to a calibrated background, but there the outcome is a rate-aware detection threshold rather than the static distinguishability object developed here.

10 Summary

The gravitational coherence surface defines an operational boundary of gravitational distinguishability for each source mode in a specified environment. It does not modify Einstein's equations. It does not conflate observability with dynamics. It does not claim a Shannon theorem where no bandwidth has been specified. And it does not claim unique recovery of arbitrary interior structure from shell shape alone.

What it offers is still substantial:

1. mode-resolved geometry for the loss of external readability, with a single-crossing condition ensuring well-posed shells and conditional nesting criteria for ordinary sources;
2. environmental dependence, making coherence a relational rather than intrinsic property;
3. an inversion formula for the observable exterior multipole spectrum, with uncertainty propagation scaling as $(l+3)$;

4. a static distinguishability functional quantifying the visibility budget at each distance, together with an effective class count meaningful for model selection;
5. clean connections to known influence-boundary families and a consistency check furnished by the Earth–Moon system; and
6. a concrete observational program centered on environment-dependent recoverability.

The Earth-Moon example remains the clearest illustration of the central distinction: dynamical belonging and gravitational readability can part ways. If that distinction survives contact with simulations and data, the coherence surface may become a useful organizing object across celestial mechanics, relativistic geodesy, and galaxy-scale inference.

In plain language, the formalism is best understood not as new gravity, but as a disciplined bookkeeping geometry for when a source-specific gravitational signal stops standing out from the surrounding gravitational background.

Appendix A. Reproducibility Notes for the Worked Examples

This appendix is intentionally modest. The examples in the main text are order-of-magnitude demonstrations of the formalism, not catalog-quality parameter inferences. The goal here is to make the worked numbers regenerable without pretending to a precision that the simplified background model does not support.

A.1 Common Conventions

Unless otherwise stated,

A.2 he monopole channel uses the acceleration criterion $r_0 = (GM/a_{\text{eff}})^{1/2}$;
igher channels use $r_l = (\alpha_l G Q_l / N_{\text{eff}})^{1/(l+3)}$ with $\alpha_l = 1$ as the default order-unity angular coefficient;

- $l_{\max}(r)$ is the highest modeled channel satisfying $\rho_l(r) > 1$;
- the distinguishability functional is evaluated as

$$D(r) = \sum_l g_l \max[\log_2 \rho_l(r), 0], \qquad g_0 = 1, \quad g_l = 2l + 1 \ (l \geq 2);$$

- all quoted shells neglect N_{inst} and N_{model} unless explicitly discussed, so $N_{\text{eff}} \to N_{\text{env}}$ and $a_{\text{eff}} \to a_{\text{bg}}$.

10.2 Dominant-Neighbor Background Choices

Table 4: Dominant-neighbor assumptions used for the main worked examples. These are deliberately simple background choices; a production analysis would replace them with direction-dependent environmental reconstructions.

Source system	Monopole background	Tidal background	Adopted separation scale	Role in paper
Sun	Proxima-like neighbor, $M' \sim 0.12 M_\odot$	Same dominant neighbor	$D \approx 1.30$ pc	Local stellar neighborhood illustration
Earth	Sun at 1 AU	Moon at 384,000 km	1 AU / 3.84×10^8 m	Earth–Moon conceptual check
Jupiter	Sun at 5.2 AU	Sun at 5.2 AU	5.2 AU	Solar-system giant-planet example
Milky Way	Andromeda-like neighbor, $M' \sim 1.5 \times 10^{12} M_\odot$	Same dominant neighbor	$D \approx 770$ kpc	Local Group illustration
$10^6 M_\odot$ globular cluster	Milky Way disk or void environment	not separately modeled	8 kpc / 5 Mpc	Environmental-scaling example

For the Sun, the corresponding dominant-neighbor tidal floor is

$$N_{\rm env} \approx \frac{GM'}{D^3} \approx 4.5 \times 10^{-31}\,{\rm s}^{-2},$$

which is the value used in the illustrative solar distinguishability table in Section 5.4. For Earth, the Sun sets the monopole background while the Moon sets the dominant tidal background; that split is essential to reproducing the Moon-outside-the-shell result without confusing acceleration and tidal criteria.

A.3 Multipole Proxies

The worked examples use simple proxies for Q_l rather than full harmonic reconstructions.

- **Earth and Jupiter.** Even zonal moments are represented schematically by $Q_l \sim MR^l|J_l|$, with the coherence radii interpreted as order-of-magnitude scales for oblateness ($l = 2$) and deeper structure ($l = 4$).
- **Milky Way.** The $l = 2$ channel is treated as a disk-dominated quadrupole proxy of the form $Q_2 \sim M_d R_d^2$, intended only to estimate the recoverability scale of non-spherical disk structure in the Local Group background.
- **Sun.** The $l = 2, 4, 6$ entries in the illustrative distinguishability table are schematic channel amplitudes chosen to show how $D(r)$ contracts with radius. They are not claimed as a precision helioseismic or solar-figure forecast.

A more complete implementation would compute Q_{lm} directly from a density model or observationally inferred harmonic decomposition, propagate directional dependence into α_l, and treat

$N_{\mathrm{env}}(\hat{n})$ as anisotropic.

A.4 How the Distinguishability Table Is Evaluated

For any chosen radius r:

1. evaluate the modeled source amplitudes $T_l(r)$ or $a_0(r)$;
2. divide by the adopted background floor to form $\rho_l(r)$;
3. retain only channels with $\rho_l > 1$;
4. compute l_{max} as the largest retained channel; and
5. sum the weighted logarithmic margins to obtain $D(r)$.

Thus the entries in the solar distinguishability table in Section 5.4 should be read as *visibility-budget bookkeeping* for a specific modeled channel set, not as immutable observables.

A.5 Uncertainty Discipline

The formal logarithmic uncertainty relation

$$\delta \ln |Q_l| = (l+3)\,\delta \ln r_l + \delta \ln N_{\mathrm{eff}} + \delta \ln \alpha_l$$

should be treated as the leading-order warning label for any empirical use of the framework. In practice, the dominant systematics will often come from environmental calibration rather than from the shell-finding step itself. That is why the observational program in Section 8 is framed around controlled comparisons rather than one-off detections.

11 References

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